Open Day of Doctoral Studies in Mathematics

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Mathematical physics and ergodic theory of non-periodic structures

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Fundamental Open Problem

Does there exist a non-periodic Gibbs measure which is
a small perturbation of a non-periodic ground-state measure
of a lattice-gas model with translation-invariant finite-range interactions?

David Hilbert 1862 - 1943

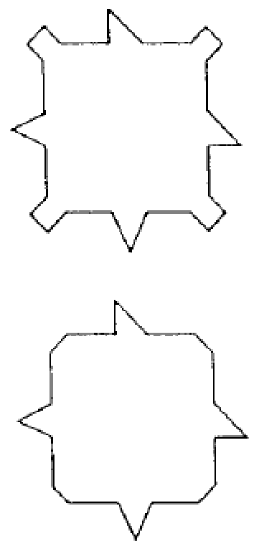
23 problems, 1900

Problem 18 Part II

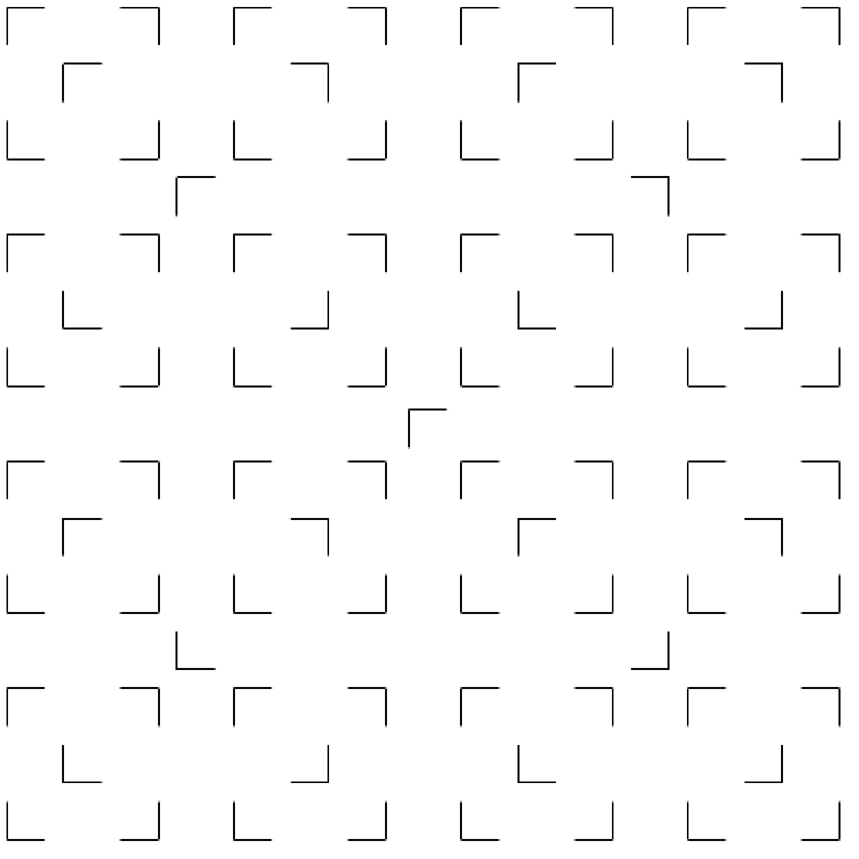
Does there exist a polyhedron which can cover the space
but only in a nonperiodic way?

Raphael Robinson 1911 - 1995

6 (56) tiles which cover planes but only in a non-periodic way, 1971



Structure of an infinite tiling



Configurations with period 2^{n+1} on sublattices $2^n Z^2$ $n \geq 1$

Global order from local rules

Classical lattice-gas models based on tilings

tiles $\rightarrow$ particles

matching rules $\rightarrow$ interactions

If two tiles do not match, then the energy of interaction between corresponding particles is positive, say 1, otherwise the energy is zero

ground-state configurations – configurations which minimize the energy

forbidden patterns have positive energy

tilings $\rightarrow$ ground-state configurations

ground-state configurations have zero energy

Systems of finite type

$$\Omega = \{1, ..., 56\}^{Z^2}$$

Let X be a Robinson's tiling, $X \in \Omega$

$$R = closure(\{T_a X, a \in Z^2\})$$

$$\mu_R = lim_{\Lambda \to Z^2} \frac{1}{|\Lambda|} \Sigma_{a \in \Lambda} \delta_{T_a} X$$

R is defined by a finite number of forbidden patterns – two neighboring tiles that do not match.

(R, T, μ_R) is a dynamical system of finite type

One-dimensional non-periodic systems

Thue-Morse and Fibonacci sequences

Question: how far are we from systems of finite type?

Thue-Morse sequences

substitutions

0 → 01
1 → 10

0

01

0110

01101001

0110100110010110

Let X be a Thue-Morse sequence, $X \in \{0,1\}^Z$

$$TM = closure(\{T_a X, a \in Z\})$$

$$\mu_{TM} = lim_{\Lambda \to Z} \frac{1}{|\Lambda|} \Sigma_{a \in \Lambda} \delta_{T_a} X$$

$$\lim_{k \to \infty} \frac{1}{k} \sum_{i=0}^{k-1} f(T^i(X)) = \int f d\mu_{TM}$$

(TM, T, μ_{TM}) is a uniquely ergodic dynamical system, Michael Keane, 1968

Characterizations of Thue-Morse sequences

Goal: looking for the minimal set of forbidden patterns

Gottschalk and Hedlung, 1964

TM is uniquely characterized by the absence of BBb,

where B is any word and b is its first character, 0 or 1

TM is uniquely characterized by the absence of infinitely many 4-point patterns
(Gardner, Radin, Miękisz, van Enter, 1989)

Low-temperature stability, Gibbs measures

$$X \in \{1, ..., n\}^{\Lambda}, \ \ \Lambda \subset Z^d, \ \ \Lambda \ \ finite$$

$$\rho_{\Lambda}(X) = \frac{e^{-\frac{H(X)}{T}}}{Z_{\Lambda}}$$

$$\rho_{\Lambda} \to \{Gibbs \ \ Measures\} \ \ as \ \ \Lambda \to Z^d$$

Main themes of this PhD project

Stability of ergodic ground-state measures
against perurbations of interactions between particles

Stability of ergodic ground-state measures
against thermal motions of particles

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Thank you for your attention

https://www.mimuw.edu.pl/~miekisz/