Mathematical analysis of hydrodynamic models - nonlinearities, non-locality, domain, scales

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About me

- PhD in 2013, habilitation in 2020
- work at IMPAS since 2012
- postdocs at University Denis Diderot (Paris), Imperial College London, Academy of Sciences of Czech Rep.
- research interest – you will see below
- awrob@impan.pl
- https://www.mimuw.edu.pl/ aw214690/Projects
- personally: mum of 2

Hydrodynamic models - systems of PDEs

- Mathematical analysis of nonlinear Partial Differential Equations (PDEs) and their solutions
- Problems (systems of equations) are related to fluid mechanics, phenomena observed in natural sciences and technology

- Collective motion – movement of a large number of individuals, organisms, particle of similar shape and size interacting with each other through certain forces (preferences) adjusting their mutual position, moving with accordance with others.
- Fluid-particle interaction – dynamics of two components: particles and fluid describes the process of falling of a solid particles suspended in the liquid as a result of gravity and inertia, sedimentation phenomena or the behaviour droplets of the spray
- Astrophysics - evolution of supernovas, atmospheric flow

PDEs – What is new? General form of $\mathbf{S}$, Ω_t, $\gamma > 1$.

- Collective motion – generalized Navier-Stokes-Vlasov system

$$
\begin{aligned}
\partial_t f + \boldsymbol{u} \cdot \nabla_x f + \nabla_v \cdot ((\boldsymbol{u} - \mathbf{v}) f) &= 0, \\
\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla_x) \boldsymbol{u} + \nabla_x p - \mathrm{div}_x \mathbf{S} &= - \int_{\mathbb{R}^3} (\boldsymbol{u} - \mathbf{v}) f \, \mathrm{d}\mathbf{v}, \\
\mathrm{div}_x \boldsymbol{u} &= 0.
\end{aligned}
$$

- Fluid-particle interaction – generalized Navier-Stokes-Smoluchowsky system

$$
\begin{aligned}
\partial_t \varrho + \mathrm{div}_x (\varrho \boldsymbol{u}) &= 0, \\
\partial_x (\varrho \boldsymbol{u}) + \mathrm{div}_x (\varrho \boldsymbol{u} \otimes \boldsymbol{u}) + \nabla_x (p + \eta) - \mathrm{div}_x \mathbf{S}(\nabla_x \boldsymbol{u}) &= -(\eta + \beta \varrho) \nabla_x \Phi, \\
\partial_t \eta + \mathrm{div}_x (\eta (\boldsymbol{u} - \nabla_x \Phi)) - \Delta \eta &= 0
\end{aligned}
$$

- Evolution of supernovas - low Mach number

$$
\begin{aligned}
\partial_t \varrho + \mathrm{div}_x (\varrho \boldsymbol{u}) &= 0, \\
\partial_t (\varrho \boldsymbol{u}) + \mathrm{div}_x (\varrho \boldsymbol{u} \otimes \boldsymbol{u}) + \frac{1}{\varepsilon^2} \nabla_x (\varrho \theta)^\gamma &= \varepsilon^\alpha \mathrm{div}_x \mathbf{S}(\nabla_x \boldsymbol{u}) + \frac{1}{\varepsilon^2} \varrho \nabla_x F, \\
\partial_t (\varrho \theta) + \mathrm{div}_x (\varrho \theta \boldsymbol{u}) &= 0.
\end{aligned}
$$

… nonlinearities, non-locality, domain, scales

- Our goal here is to include non-Newtonian fluid rheology – viscosity of the fluid may change under various stimuli (shear rate, electric field)
- Time dependent domain - domain does not have to be fixed
- Low Mach number limit - incompressible limit

- nonlinearity - non-Newtonian fluids - viscosity is nonlinear function of shear rate, polynomial or even more general
- domain - domain may change it shape in time
- non-locality - alignment, adhesion, visual or sensory interaction
- scales - when some parameters start to be negligible (low Mach number)

General questions we would like to ask and answer

- Do considered systems of equations poses solutions in certain classes: weak, measure-valude, strong?
- Are those solutions global in time, unique?
- How do they behave for large times? Do they stabilise in some sense?
- Can we expect periodic solutions?
- How do particular system behave when some parameters pass to zero or infinity?

Mathematical analysis will give better understanding of some complex behaviours of considered systems, which here are associated with physical, biological and sociological models.

Methodology (in general)

We will use (possibility to learn) advanced methods of

- nonlinear partial differential equations, theory of weak solutions and measure-valued solutions,
- harmonic analysis, functional analysis, measure theory
- functional spaces, theory of Lebesgue, Sobolev, Orlicz, and Musielak-Orlicz spaces.
- penalisation methods, renormalization techniques
- compensated compactness theory, relative entropy methods

Accomplishment - local and international cooperation

The above research is a part of the project:

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nonlinearities, non-locality, domain, scales

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PhD student (student of doctoral school) involved in the project will have possibility to :

- deep the knowledge and develop research in frame of (around) the presented projects
- work in international community (Oxford University, Imperial College London, L'Aqiulla University, Academy of Sciences of the Czech Rep., Yonsei University)
- receive an additional scholarship (2 500 PLN per month)
- additional funding for participation in workshops, scientific schools, conferences, international visits

Thank you for your attention!